MATH5453M Foundations of Fluid Dynamics

Example Sheet 1

To hand in by Friday 24 October 2025 at 5pm in the box outside my office (11.07)

1. A fluid flow is given by

$$\mathbf{u} = (U_0, \sin \Omega t, 0)\,.$$

(i) Find the equation of the streamlines at time t.

(ii) Find the path of a particle that is located at (x_0, y_0, z_0) at time $t = 0$. Hence find the equation of the particle path of a particle released at the point $(0, 1, 0)$ at $t = 0$.

(iii) For $t > 0$, dye is continously released into this flow at the point $(0, 1, 0)$. Find the equation of the resulting streakline at time $t = 2\pi/\Omega$.

(iv) Sketch the streakline and the particle paths of a particle released at the point $(0, 1, 0)$ at $t = 0$. Are they different? Why?

2. An incompressible fluid occupies the space $0 \le y \le \infty$ above a plane rigid boundary which oscillates in the x-direction with velocity $U \cos(\omega t)$. We assume that the fluid velocity field has the form $\mathbf{u} = u(y, t)\hat{\mathbf{x}}$ and that the pressure is constant, $p = p_0$.

(i) Simplify the Navier–Stokes equation.

(ii) By seeking a solution of the form $u = Re\{f(y)exp(i\omega t)\}$, where $Re\{\cdot\}$ denotes the real part of $\cdot$, find the real form of the velocity u. Hint: $2i = (1 + i)^2$.

(iii) We consider the above problem for water (viscosity $\mu = 10^{-3}$ Pa·s, density $\rho = 10^3$ kg·m^{-3}), an oscillation period of 1s and $U = 1$m·s^{-1}. What is the maximum amplitude of the velocity one centimetre above the oscillating boundary?

3. We consider the steady flow

$$\mathbf{u} = -\frac{1}{2}\alpha R\hat{\mathbf{R}} + u_\phi \hat{\boldsymbol{\phi}} + \alpha z \hat{\mathbf{z}},$$

where $\hat{\mathbf{R}}$ is the unit radial vector, $\hat{\phi}$ is the unit azimuthal vector, $\hat{\mathbf{z}}$ is the unit axial vector and where R denotes the radial distance from the origin, u_ϕ may depend on R, z is the axial coordinate and where α is a constant.

(i) Show that the flow is incompressible.

(ii) Find the expression of the vorticity as a function of u_ϕ, R and α.

(iii) From the ϕ-component of the Navier–Stokes equation, obtain an equation for the vorticity.

(iv) Calculate the vorticity, where ω_0 is the vorticity at the origin.

(v) Sketch the flow in the (R, z)-plane.

(vi) Write the vorticity equation and explain the role of each of the terms.

(vii) For the above vortex and using the vorticity equation, show what the dominant phenomena are for small R.

(viii) Same question as (vii) but for large R.