Faster than Hamilton!

Optimizing Formula 1 strategies

October 14, 2024

1 Introduction

We make many decisions every day, from what we choose for breakfast to when we decide to go back to sleep. Our lives are built on decisions with nontrivial consequences and we often have to improvise. Modelling provides a clever way to build intuition and experience, and to optimise the outcome of our choices. Decision making is at the core of professions such as stock trader or CEO but it is also hidden within unsuspicious tasks (and less obvious professions) such as the prioritisation of tasks or the choice of tool to complete a task. Here is a directly relatable example: a (bad) student follows a 15-credit module and a 10-credit module. The exams are in four days and they have not started revising. How should they plan their revision time?

In this project, we consider a simple model of an extreme profession: Formula 1 Strategy Engineer. In Formula 1, each driver must complete a certain number of laps as quickly as they can. To improve their pace, they can stop to put on fresher tyres and refuel the car, allowing them to carry less fuel at once (note that refuelling is currently forbidden). While planning their race, they have to take into account a large number of parameters which will impact their performance, such as tyre wear and fuel load [1]. The problem is too complicated to handle for the drivers who have the demanding task to drive the car to its limits, so Formula 1 teams rely on Strategy Engineers. Ruth Buscombe (Figure 1), former Strategy Engineer for the Formula 1 team Sauber, commented on her work in a video interview [2].

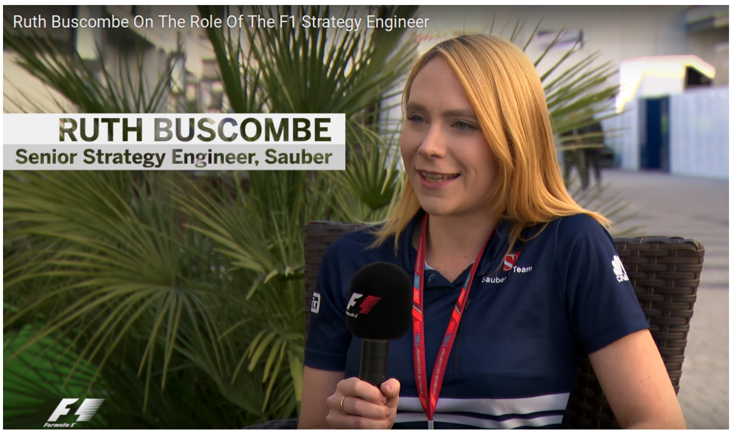

Figure 1: Ruth Buscombe was a Formula 1 Strategy Engineer for the Sauber F1 Team. After [2].

Since all decisions depend on the level of performance of the car, the most important task of the Strategy Engineer is to reliably predict lap times. This is achieved using a lap time modeller similar to that introduced in Section 2. Most race decisions relying on whether or not to change tyres and refuel and when, we introduce the concepts of pit-stop and optimum strategy in Section 3. Section 4 provides a list of example questions for your project. Lastly, in Section 5, we introduce a competition to test your understanding of race strategy. Will you come out on top?

2 Lap time modelling

2.1 Hypothesis

A good first approximation is to assume that the lap time is only affected by tyre wear and fuel load: the more a set of tyres is used, the less performant; the more fuel onboard, the heavier and slower the car. Neglecting any nonlinear coupling, we can write:

$$t = t_b + p_t + p_f, \tag{1}$$

where t is the lap time, t_b is the base time, corresponding to the perfect/fastest possible lap, p_t is the time penalty due to tyre wear and p_f is the time penalty due to the fuel load onboard. Coming up with the laws associated with these quantities is the job of the Strategy Engineer and involves collecting data on past races and collapsing them onto simple formulae.

2.2 Example

Let us assume that the Strategy Engineer finds the following laws for a circuit:

$$t_b = 80\text{s}, \tag{2}$$

$$p_t(l) = T\,l, \tag{3}$$

$$p_f(l, l_{max}) = F\,(l_{max} - l), \tag{4}$$

where $T > 0$ is the additive time penalty per lap due to tyre wear, $F > 0$ is the additive time penalty per lap due to the weight of fuel onboard, l is the number of laps already completed during the current stint and l_{max} is the number of laps of fuel onboard at the start of the stint. Lap time model (1) becomes:

$$t = 80 + F\,l_{max} + (T - F)l. \tag{5}$$

If we further assume that the tyre and fuel penalties account for 1 second per lap each, $T = F = 1$s, we have:

$$t = 80 + l_{max}. \tag{6}$$

For a 4-lap stint, the total racing time is thus:

$$t_{stint} = \sum_{l=1}^{l=l_{max}} t = 4(80 + 4) = 336\text{s}, \tag{7}$$

so that the driver completes the stint in 336s.

3 Optimum strategy

3.1 Pit-stops

During a pit-stop, the driver stops the car to change tyres and refuel. This allows the car to be lighter during most of the race (it does not have to carry the fuel load necessary to complete the race from the start) and to wear fresher, and thus faster, tyres. Both of these account for an increased performance. There is, thus, a tradeoff between the time lost pitting and the time gained by running a lighter car on fresher tyres.

Calculating the optimum number of pit-stops and when to make them is an optimization problem that can win races. Figure 2 shows the 2016 Abu Dhabi Grand Prix race report by tyre manufacturer Pirelli. The drivers were allowed to use three tyre compounds, from hardest (slowest) to softest (fastest): Soft,

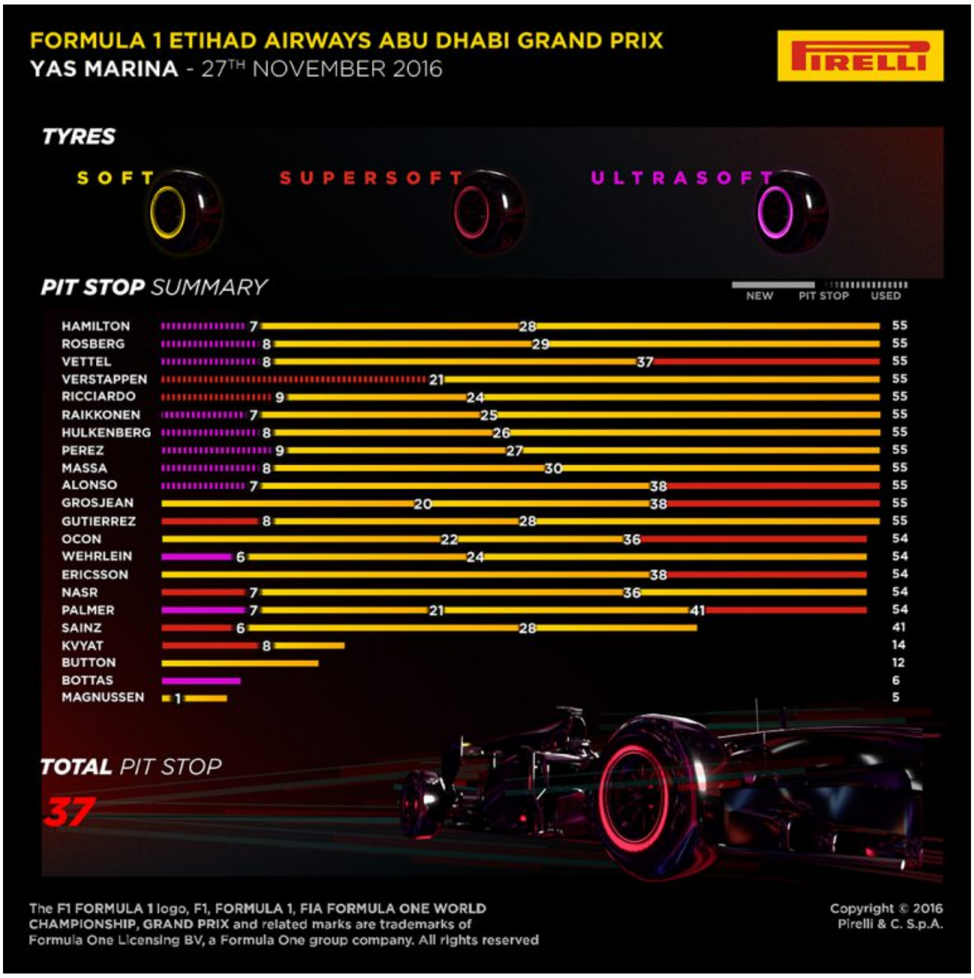

Figure 2: Pirelli race report from the 2016 Abu Dhabi Grand Prix showing the pit stop strategy used by each driver. Soft tyres are shown in yellow and are the hardest compound, followed by Supersoft tyres, in red and Ultrasoft tyres, in purple. After [9].

Supersoft and Ultrasoft. Several strategies were used during the race: Hamilton and Rosberg opted for a two-stop strategy using Ultrasoft, then Soft and finally Soft tyres; Vettel and Alonso made the second stint longer and used Supersoft tyres in their last stint; Verstappen opted for a one-stop strategy with Supersoft and Soft tyres; etc.

3.2 Example

Let us consider a 4-lap race, assume that a pit-stop costs $t_p = 7$s and only consider one pit-stop strategies. There are only three pit-stop possibilities: lap 1, lap 2 and lap 3. Let us calculate the race time for each of these strategies.

3.2.1 Pitting on lap 1

If we decide to pit on lap 1, the total race time becomes:

$$t_{race} = t_{stint}(l_{max} = 1) + t_p + t_{stint}(l_{max} = 3), \tag{8}$$

where $t_{stint}(l_{max})$ is the stint time corresponding to a stint of l_{max} laps. Here, we have:

$$t_{stint}(l_{max} = 1) = \sum_{l=1}^{l=l_{max}} t = 81\text{s}, \quad (9)$$

$$t_{stint}(l_{max} = 3) = \sum_{l=1}^{l=l_{max}} t = 249\text{s}, \quad (10)$$

such that the first stint (until the pit-stop) lasts 81s and the second stint (between the pit-stop and the end of the race) lasts 249s. The race time is then $t_{race} = 81 + 7 + 249 = 337$s, which is longer than the 336s obtained from the zero pit-stop strategy in Section 2.2.

3.2.2 Pitting on lap 2

If we decide to pit on lap 2, the total race time becomes:

$$t_{race} = t_{stint}(l_{max} = 2) + t_p + t_{stint}(l_{max} = 2), \quad (11)$$

with:

$$t_{stint}(l_{max} = 2) = \sum_{l=1}^{l=l_{max}} t = 164\text{s} \quad (12)$$

so that the total race time is $t_{race} = 2 * 164 + 7 = 335$s, which is shorter than the 336s obtained from the zero pit-stop strategy.

3.2.3 Pitting on lap 3

Due to obvious symmetries, this calculation is the same as that in Section 3.2.1 and we obtain 337s, which is slower than the zero pit-stop strategy by 1s.

3.2.4 Conclusion

For this specific example, the one-stop strategy is the fastest provided that the pit-stop is made on lap 2. The driver will then complete the race in 335s. The zero pit-stop strategy is 1s slower while the worst strategies were one pit-stop strategies with the pit-stop either on lap 1 or on lap 3.

4 Questions

(Data1) Obtain a database of lap times and find the corresponding value for T and F. Hint: You can find such databases online, e.g. [4, 5].

(Data2) Are the linear laws (3) and (4) good approximations for the tyre wear and fuel load effects? If not, can you come up with a better fit?

(Data3) Are there other effects that would modify equation (1)?

(Strat1) The discrete summations are an inefficient way to calculate the race time (equation (7)) or stint time (equations (9)–(12)). Could you come up with a better formulation for these calculations? Hint: Some help can be obtained in [8].

(Strat2) How can we model the duration of a pit-stop t_p?

(Strat3) Assume you enter a 10-lap race and consider the lap time model you obtained in Questions (Data1)–(Data3) and the pit-stop duration you obtained in Question (Strat2). What is the best strategy, i.e., how many pit-stops should you make and when?

(Strat4) How sensitive is this strategy to changes in the parameter values (pit-stop duration, tyre wear effect, fuel load effect)?

(Strat5) Same question as Questions (Strat3) and (Strat4) but for an N-lap race, $N > 10$.

5 Let's compete!

Participation is mandatory. Intellectual effort will be marked positively but performance will not be assessed. The competition documentation is available at:
`http://cbeaume.com/download/F1StratCompetition.pdf`.

6 Some other interesting references

Here is a short and non-exhaustive bibliography guide:

- A 2011 review on prominent articles regarding the physics of sport [6]
- A mechanistic approach to modelling vehicle handling, complemented with experiments [10]
- A lap time model not too dissimilar to ours [7]
- On decision-making in racing [3]
- Using machine-learning techniques to inform decision makers in racing [11]

References

[1] Formula 1. Race strategy.
`https://www.formula1.com/en/championship/inside-f1/...`
`...understanding-f1-racing/Race_strategy.html`.

[2] Formula 1. Ruth buscombe on the role of the f1 strategy engineer. *Youtube*, 2017.
`https://www.youtube.com/watch?v=K7EEBE-2Wf4`.

[3] J. Bekker and W. Lotz. Planning Formula One race strategies using discrete-event simulation. *J. Operational Res. Soc.*, 60:952–961, 2009.

[4] J. Bouchet and E. Wojcikiewicz. Formula 1 race data. 2017.
`https://www.kaggle.com/cjgdev/formula-1-race-data-19502017`.

[5] K. Collantine and D. Rencken. Racefans independent motorsport coverage. 2018.
`https://www.racefans.net/`.

[6] C. Frohlich. Resource Letter PS-2: Physics of Sports. *Am. J. Phys.*, 79:6, 2011.

[7] A. Heilmeier, M. Graf, J. Betz, and M. Lienkamp. Application of monte carlo methods to consider probabilistic effects in a race simulation for circuit motorsport. *Appl. Sci.*, 10:4229, 2020.

[8] W. Mulholland. Formula one race strategy. *Royal Academy of Engineering*.
`https://www.raeng.org.uk/publications/other/14-car-racing`.

[9] Pirelli. Pit stop summary from the 2016 abu dhabi grand prix.
http://www.f1strategyreport.com/2016/11/29/...
...f1-strategy-report-abu-dhabi-grand-prix-2016/.

[10] B. Siegler and D. Crolla. Lap time simulation for racing car design. *SAE Transactions*, 111(7):306–314, 2002.

[11] T. Tulabandhula and C. Rudin. Tire changes, fresh air and yellow flags: challenges in predictive analytics for professional racing. *Big Data*, 2:97–112, 2014.