Spatial localization: recent progress in theory and applications

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Spatially localized structures occur in many continuous dissipative systems and play an important role in their dynamics. They typically take the form of bound states of stationary or time-dependent fronts and have been observed in optics, biology and fluids. They are relevant to understanding the edge separating two stable states, such as laminar and turbulent shear flows. This minisymposium brings worldwide experts together to discuss and provide a summary of the recent progress in this area. The minisymposium is also of interest from a technical point of view as the investigation of spatial localization involves cutting-edge mathematical techniques.

Confirmed speakers

- **Daniele Avitabile** (University of Nottingham, UK) – daniele.avitabile@nottingham.ac.uk
  Localised solutions in integral neural field equations
- **Cédric Beaume** (UC Berkeley, USA) – ced.beaume@gmail.com
  Localized convection in a rotating fluid layer
- **Paul Bushby** (Newcastle University, UK) – paul.bushby@ncl.ac.uk
  Localised states in magnetoconvection
- **Lendert Gelens** (Stanford University, USA) – lendert.gelens@gmail.com
  Dynamics of cavity solitons and optical frequency combs in the Lugiato–Lefever equation
- **David Lloyd** (University of Surrey, UK) – d.j.lloyd@surrey.ac.uk
  Nucleation of localised Ferro-patterns
- **David Lo Jacono** (Université de Toulouse, France) – david.lojacono@imft.fr
  Moving Localized Structures in a doubly diffusive system
- **Matthew Salewski** (*formerly with* Max Planck Institute for Dynamics and Self-Organization, Germany) – matthew.salewski@ds.mpg.de
  Localized solutions in plane Couette flow with rotation
- **Björn Sandstede** (Brown University, USA) – bjorn_sandstede@brown.edu
  Stability properties of localized structures near snaking

MS65 Spatial Localization: Recent Progress in Theory and Applications Part I of II

Wednesday, August 13, 4:30 PM - 6:30 PM
Room: Jock Colville Hall - Archives Center

- Elizabeth J. Makrides & **Björn Sandstede** (Brown University, USA)
  Stability properties of localized structures near snaking
  Snaking refers to the existence of localized roll structures in spatially extended, reversible 1D partial differential equations, that exist along a vertical sine-shaped bifurcation curve so that the width of the underlying periodic roll pattern increases along the bifurcation curve. Localized rolls undergo a series of bifurcations along the branch, and this talk is concerned with analytical and numerical aspects of their PDE stability spectra.

- **Lendert Gelens** (Stanford University, USA), Pedro Parra-Rivas (Vrije Universiteit Brussel, Belgium), Damia Gomila (Universitat de les Illes Balears Palma de Mallorca, Spain), François Leo (Ghent University, Belgium) & Stéphane Cohen (University of Auckland, New Zealand)
  Dynamics of cavity solitons and optical frequency combs in the Lugiato–Lefever equation
  Kerr frequency combs can be modeled in a similar way as temporal cavity solitons (CSs) in nonlinear cavities, using the Lugiato-Lefever equation describing pattern formation in optical systems. Here, we first characterize different dynamical regimes of CSs, such as time-periodic oscillations and various chaotic dynamics. Secondly, the effect of third order dispersion on the stability and snaking structure of CSs is studied. Finally, we discuss how the dynamics change as the cavity size decreases.

- **David Lloyd** (University of Surrey, UK) & Reinhard Richter (University of Bayreuth, Germany)
  Nucleation of localised Ferro-patterns
  We present a numerical scheme for computing stationary patterns on the one- and two-dimensional surface of a magnetic fluid of finite depth using coordinate transformations to yield equations in rectan- gular domains where standard psuedo-spectral methods can be used. We show how one can implement a phase condition to eliminate the non-uniqueness of solutions due to the magnetic potentials only being defined up to a constant. The scheme is then embedded into a pseudo-arclength continuation algorithm with a Newton-GMRES solver allowing one to trace out bifurcation diagrams. We carry out a systematic parameter study of spatially periodic ridges and locate the region of existence of localised 1D/2D-interfacial patterns as the applied magnetic field strength, magnetic permeability and fluid depth are varied.

- **Matthew Salewski** (*formerly with* Max Planck Institute for Dynamics and Self-Organization, Germany) & Tobias M. Schneider (École Polytechnique Fédérale de Lausanne, Switzerland)
  Spatial localisation and developing complexity in pipe flow
  Localized solutions of plane Couette flow (PCF) bifurcate from the subcritical 3D periodic state at infinite parameter; forcing the system through rotation changes this. The 3D periodic state now emerges secondarily from a 2D periodic state, making the localized solution a 3D patch embedded in a 2D patterned background. The nontrivial background has some effect, however much of the scenario seen in other systems remains, such as the snakes-and-ladders structure already found in non-rotating PCF.

MS73 Spatial Localization: Recent Progress in Theory and Applications Part II of II

Thursday, August 14, 8:10 AM - 10:10 PM
Room: Jock Colville Hall - Archives Center

- **Daniele Avitabile** (University of Nottingham, UK)
  Localised solutions in integral neural field equations
  I will discuss the formation of stationary localised solutions in integral neural field models with inhomogeneous synaptic kernels and Heaviside firing rate functions. We consider a simple periodic modulation in the synaptic kernel, apply interface methods to the resulting integral problem and show how to construct localised bumps with a finite activity region and two threshold crossings. Localised solutions are arranged in a "snake and ladder" bifurcation diagram. However, in this context, interface methods allow for the explicit construction of a bifurcation equation for localised steady states, so that analytical expressions for snakes and ladders can be derived. Similarly, eigenvalue computations can be carried out analytically to determine the stability of the solution profiles. In addition, we find regions of parameter space where the trivial homogeneous state coexists with two other stable solutions, an above-threshold periodic state and a cross-threshold periodic state. We show how the multiple stability affects the corresponding bifurcation diagram and examine models in which the firing rate is a steep sigmoid as opposed to the Heaviside step function. This is joint work with Helmut Schmidt (Exeter)

- **Cédric Beaume** (UC Berkeley, USA)
  Localized convection in a rotating fluid layer
  We study stationary convection in a two-dimensional fluid layer rotating around the vertical and heated from below. With stress-free boundary conditions, spatially localized states are embedded in a self-generated background shear zone and lie on a pair of intertwined solution branches exhibiting "slanted snaking". Similar solutions with no-slip boundary conditions are computed. They are not embedded in a background shear and the solution branches exhibit snaking without a slant. Homotopic continuation from free-slip to no-slip boundary conditions is used to track the changes in the properties of the solutions and the associated bifurcation diagrams.

- **Paul Bushby** & Matthew Buckley (Newcastle University, UK)
  Localised states in magnetoconvection
  Previous studies have shown that it is possible to find steady, localised convective cells in two-dimensional Boussinesq magnetoconvection. Building on earlier work, we demonstrate the existence of oscillatory localised convective states in this system. We discuss the properties of these solutions as well as the effects of changing the horizontal boundary conditions. In particular, we show that the inclusion of impermeable sidewalls can lead to formation of localised "wall" modes.

- **David Lo Jacono**, Alain Bergeon (Université de Toulouse, France) & Edgar Knobloch (UC Berkeley, USA)
  Moving Localized Structures in a doubly diffusive system
  In this talk we will describe the origin and properties of moving spatially localized structures in natural doubly-diffusive convection in a vertically extended cavity. These solutions arise through secondary parity breaking bifurcations. Both single pulse and multipulse states of this type will be described. The numerical results will be related to the phenomenon of homoclinic snaking in spatially reversible systems.