Spatial localisation in fluids

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Confirmed speakers

- **Cédric Beaume** (University of Leeds, UK) – c.m.l.beaume@leeds.ac.uk
  From convectons to complexity in doubly diffusive convection
- **Yohann Duguet** (CNRS/LIMSI, France) – duguet@limsi.fr
  Oblique stripes of turbulence in plane Poiseuille flow
- **David Lo Jacono** (Université de Toulouse, France) – david.lojacono@imft.fr
  Spatially localized radiating diffusion flames
- **Isabel Mercader** (Universitat Politècnica de Catalunya, Spain) – maria.isabel.mercader@upc.edu
  Localized binary convection in a slightly inclined rectangular cavity
- **Alvaro Meseguer** (Universitat Politècnica de Catalunya, Spain) – alvaro.meseguer@upc.edu
  Creation and annihilation mechanisms of localized travelling pulses in plane shear flows
- **Yasumasa Nishiura** (Tohoku University, Japan) – yasumasa@pp.iij4u.or.jp
  Dynamics of spatially localized patterns in binary fluids

Minisymposium: Spatial localisation in fluids
Part I of III

- **Cédric Beaume** (University of Leeds, UK), Alain Bergeon (IMFT, Université de Toulouse, France) & Edgar Knobloch (UC Berkeley, USA)
  From convectons to complexity in doubly diffusive convection
  Doubly diffusive convection arises frequently in natural phenomena and industrial processes and occurs in systems characterised by competing fields that diffuse at different rates. Well-known examples are provided by thermohaline convection and the salt finger instability. In this talk, we consider three-dimensional thermohaline convection where a binary mixture is confined between vertical walls maintained at different temperatures and salinities. In this configuration, we found stationary spatially localised solutions consisting of spots of convection embedded in a background conduction state. These *convectons* are formed through a subcritical bifurcation from the conductive state (motionless fluid) and display a variety of patterns while simulations above onset reveal chaotic dynamics.

- Chaitanya S. Paranjape (IST Austria), **Yohann Duguet** (LIMSI-CNRS, Université Paris-Saclay, France) & Björn Hof (IST Austria)
  Oblique stripes of turbulence in plane Poiseuille flow
  The onset of turbulence in extended three-dimensional plane Poiseuille flow is characterised by oblique stripe patterns of turbulent motion co-existing spatially with laminar flow. We first show experimental evidence that these stripes relaminarise by themselves for low enough flow rates. Using direct numerical simulation in a reduced oblique geometry, we unfold for the first time a sequence of bifurcations leading from a simple localised (unstable) travelling wave to a strange repeller, via relative periodic orbits and unstable relative tori. This study, parametrised by the angle of the stripe, suggests a minimal value of the Reynolds number for the appearance of localised turbulent motion.

- **David Lo Jacono**, Alain Bergeon (IMFT, Université de Toulouse, France) & Edgar Knobloch (UC Berkeley, USA)
  Spatially localized radiating diffusion flames
  We study a simple model of 2D radiating diffusion flames. A large variety of stationary spatially localized states representing the breakup of the flame front near extinction is computed using numerical continuation. These states are organized by a global bifurcation in space that takes place at a particular value of the Damköhler number and their existence is consistent with current understanding of spatial localization in driven dissipative systems.

Minisymposium: Spatial localisation in fluids
Part II of III

- **Isabel Mercader**, Oriol Batiste & Arantxa Alonso (Universitat Politècnica de Catalunya, Spain)

  Localized binary convection in a slightly inclined rectangular cavity

  Thermal convection in a binary fluid layer heated from below is a system that exhibits a great variety of pattern forming phenomena when driven away from equilibrium. In binary mixtures, the concentration flux depends both on concentration and temperature gradients (Soret effect). This effect is quantified by the Soret coefficient, and its sign determines the behavior of the mixture. With a negative Soret coefficient, the heavier component migrates towards the hotter boundary, resulting in a stabilizing concentration gradient competing with the destabilizing thermal gradient that produces it. In such type of mixtures, an especially interesting stationary localized structure has been identified in rectangular cells, the so-called *convectons* [1]. The systematic study of these states has motivated many recent works among the international community. Our team has participated actively in this study, helping to understand their origin and properties, mainly with the discovery of the snaking branches of solutions where convectons are located [2–5]. Such states can be disturbed by a small inclination of the cell (by an angle θ). This effect is particularly interesting as it induces a flow at any value of the heating and, thus, prevents the existence of the no-flow conductive state found in the horizontal case. We investigate the extent to which any type of the persistent convectons can be achieved in the tilted cell as a result of the interaction between the localized structure and the basic large scale flow that substitutes the quiescent state.

  The symmetries of the governing equations change when the inclination is considered, and modifications of the bifurcation diagrams are expected. We will show the splitting of the branches of centered *odd parity convectons* (invariant under a left-right reflection followed by a reflection in the layer midplane), which are related by some of the symmetries of the problem and coincide when global quantities of the solutions are plotted (see Figure 1). We will see that they evolve in

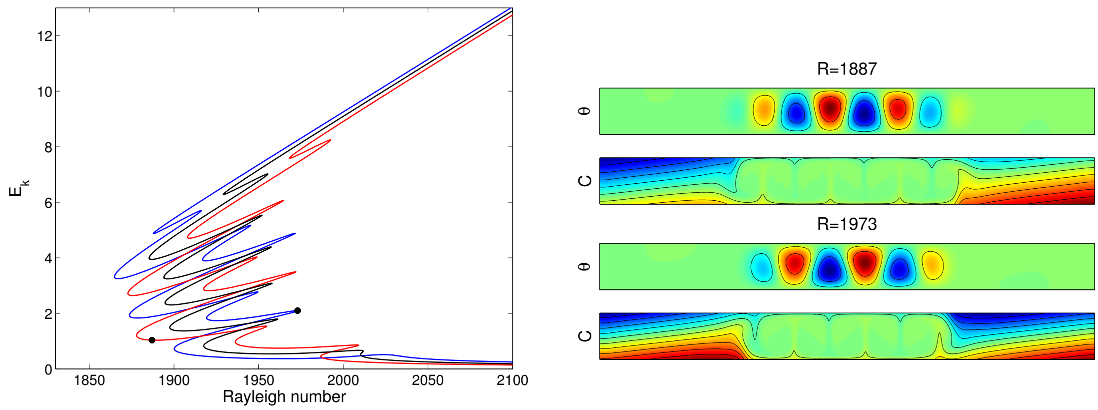

Figure 1: (left) Kinetic energy (E_k) as a function of the Rayleigh number (Ra) for the two branches of centered *odd parity convectons* for an inclination of $\theta = 0.01$ rad (red and blue), resulting from the splitting of the branch for $\theta = 0$ (black). (right) Deviation of the temperature with respect to a vertical linear profile and concentration field for two convectons at the points denoted by a solid circle in the (Ra- E_k) diagram.

  a different way when the inclination increases. Centered *even parity convectons* (symmetric under left-right reflection) become asymmetric localized states.

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Minisymposium: Spatial localisation in fluids
Part III of III

- **Alvaro Meseguer** & Fernando Mellibovsky (Universitat Politècnica de Catalunya, Spain)
  Creation and annihilation mechanisms of localized travelling pulses in plane shear flows
  In this talk we will study different localization and delocalizations mechanisms of travelling waves in plane channel flows. One of the mechanisms consist of subharmonic Hopf bifurcations of replicated Tollmien–Schlichting Waves (TSW) in arbitrary long domains. We will show how these bifurcations sometimes exhibit Benjamin–Feir instabilities leading to localized modulated travelling pulses. We will show how these instabilities may also lead to modulated travelling waves that do not necessarily exhibit localization but serve as connection between upper and lower branches of TSW. Finally we will see that localized pulses may exhibit snaking scenarios that are not always robust, being structurally very sensitive to changes of length channel.

- Takeshi Watanabe (Japan Aerospace Exploration Agency, Japan), Makoto Iima (Hiroshima University, Japan) & **Yasumasa Nishiura** (Tohoku University, Japan)
  Dynamics of spatially localized patterns in binary fluids
  Binary fluid is a mixture of two miscible fluids such as water and ethanol. When a temperature gradient is applied to a binary fluid, "Soret effect", an effect where the temperature gradient induces a concentration gradient, appears. There are both positive and negative Soret effects. In the positive case, the heavier component is condensed in cooler parts and vice versa. Our work focused on the negative effect, in which case, the temperature field activates the convection, which is in turn inhibited by the concentration field. In contrast to monofluid, there appears spatially localized convection patterns such as steady pulse (SP, also known as "convecton") and time-periodic traveling pulse (PTP). We discuss about the collision dynamics between two PTPs. A variety of patterns appear after the complex transition process when counter-propagating PTPs collide with each other. In these transition processes, even-symmetric SPs play a role as "separators" and thus the network of saddles consisting of even-symmetric SPs is very important to predict the asymptotic state of collisions. We present this network structure as well as a detailed analysis of the even-SP branch.