Dr. C. Beaume

MATH3620 Fluid Dynamics 2

Example sheet 3

1. Estimate the Reynolds number for the following situations:

   (a) A raindrop of size 0.5cm falling through air at $5\text{m}\cdot\text{s}^{-1}$. Viscosity of air is approximately 2×10^{-5}Pa·s and density, $1.3\text{kg} \cdot \text{m}^{-3}$.

   (b) A spermatozoan with tail length 10μm swimming at $100\mu\text{m}\cdot\text{s}^{-1}$ in water. Viscosity of water is approximately 10^{-3}Pa·s and density, $10^3\text{kg} \cdot \text{m}^{-3}$.

   (c) A jet of ink from an inkjet printer: nozzle diameter 50μm, jet speed $5\text{m}\cdot\text{s}^{-1}$ for an ink of density $10^3\text{kg} \cdot \text{m}^{-3}$ and viscosity 0.01Pa·s.

   (d) Pouring honey from a jar: viscosity 10Pa·s, density $1.3 \times 10^3\text{kg} \cdot \text{m}^{-3}$.

   (e) Convection in the earth's mantle: depth 700km, moving at 20mm per year. Viscosity of mantle is approximately 10^{22}Pa·s and density, $4 \times 10^3\text{kg} \cdot \text{m}^{-3}$.

2. Show from the Stokes equations that, when $Re = 0$, the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ satisfies:

$$\nabla^2 \boldsymbol{\omega} = 0.$$

   Show that the streamfunction ψ for a planar flow is related to the vorticity by:

$$\nabla^2 \psi = -\omega,$$

   and show that the streamfunction for a planar Stokes flow satisfies:

$$\nabla^4 \psi = 0,$$

   where $\nabla^4 f = \nabla^2(\nabla^2 f)$.

3. Estimate the size of $\rho \frac{D\mathbf{u}}{Dt}$ in the slider bearing flow. Hence, show that the effects of inertia can be neglected provided that:

$$U \ll \frac{2\mu L}{\rho |d_2^2 - d_1^2|}.$$

   Deduce that the lubrication approximation remains valid even for flows for which the Reynolds number, $\frac{\rho U(d_1+d_2)}{2\mu}$, is not small.

4. A slow viscous flow is driven through a narrow gap between two rigid walls at $y = 0$ and $y = h(x)$ by an applied pressure gradient. Show that, provided $\frac{dh}{dx} \ll 1$, the pressure p is approximately constant in the cross-section and that the volume flow rate (per unit length in z):

$$Q = \int_0^{h(x)} u dy = -\frac{h^3}{12\mu}\frac{dp}{dx}.$$

Find the pressure difference between $x = 0$ and $x = L$ required to produce a volume flow rate of Q for the case:

$$h(x) = d_1 + \frac{(d_2 - d_1)}{L}x.$$

5. A thin layer of fluid of viscosity μ and density ρ is draining under the effect of gravity down the side of a vertical plane. Its thickness, $h(z,t)$, is only a function of time t and vertical distance z down the plane. State the conditions under which the lubrication approximation is valid and show that, in this case, $h(z,t)$ follows the evolution equation:

$$\frac{\partial h}{\partial t} + \frac{\rho g h^2}{\mu}\frac{\partial h}{\partial z} = 0.$$

Verify that:

$$h(z,t) = f\left(z - \frac{\rho g h^2}{\mu} t\right),$$

is a solution of this equation. What does this solution tell you about how the shape of a bulge evolves?

Please send any comments or corrections to Dr. C. Beaume.
c.m.l.beaume@leeds.ac.uk