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MATH3620 Fluid Dynamics 2

Example sheet 2

1. The gap between two parallel walls at $y = 0$ and $y = h$ is filled with a fluid of viscosity μ and density ρ. A pressure gradient $\frac{\partial p}{\partial x} = -G$ is applied along the channel with the wall at $y = h$ held fixed while the wall at $y = 0$ has velocity $-U$ in the x direction. Show that the steady fluid velocity is given by:

$$u(y) = \frac{G}{2\mu} y\,(h - y) - \frac{U}{h}\,(h - y)\,.$$

Calculate the volume flow rate:

$$Q = \int_0^h u(y) dy,$$

and show that if $G = 6\mu U/h^2$ then $Q = 0$. For this case, find the maximum value of u in the channel and show that $u(h/3) = 0$. Sketch the velocity profile across the gap.

2. The gap between two parallel walls at $y = -h$ and $y = h$ is filled with two different immiscible fluids, such that the fluid in $-h < y < 0$ has viscosity μ_1, while that in $0 < y < h$ has viscosity μ_2.

   (a) State the boundary conditions that must be applied at $y = 0$.

   (b) If the boundary at $y = h$ has velocity U, the one at $y = -h$ has velocity $-U$ and there is no applied pressure gradient, find the fluid velocity in the gap. (Hint: treat the velocity at $y = 0$ as an unknown and find the fluid velocity in each fluid separately, then apply the boundary conditions at $y = 0$).

   (c) The boundaries are now fixed and a pressure gradient $\frac{\partial p}{\partial x} = -G$ is applied. Find the fluid velocity in the two fluids. Calculate the volume flow rates Q_1 and Q_2 of each fluid and show that:

$$\frac{Q_1}{Q_2} = \frac{\mu_2(7\mu_1 + \mu_2)}{\mu_1(7\mu_2 + \mu_1)}.$$

3. A fluid of viscosity μ is forced to flow in the gap between two cylinders at $r = a$ and $r = b$ by a pressure gradient $\frac{\partial p}{\partial z} = -G$. Find the fluid velocity on the assumption that it is of the form $\mathbf{u} = w(r)\hat{\mathbf{e}}_{\boldsymbol{z}}$. Hence determine the shear stress at $r = a$ and $r = b$.

4. A viscous fluid is forced to flow down a pipe of constant cross-section S whose axis runs parallel to the z axis by a pressure gradient $\frac{\partial p}{\partial z} = -G$.

(a) Show that the fluid velocity, $\mathbf{u} = w(x, y)\hat{\mathbf{e}}_{\boldsymbol{z}}$, must satisfy:

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{G}{\mu},$$

subject to the boundary condition $w = 0$ on the boundary ∂S.

(b) By seeking a solution of the form:

$$w(x, y) = A\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right),$$

find the velocity in the case S is the interior of the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

and show that it reduces to the Poiseuille flow solution when $a = b$.

(c) Now consider the case in which S is the interior of an equilateral triangle with sides $y = 0$, $y = \sqrt{3}(a - x)$ and $y = \sqrt{3}(a + x)$. Show that:

$$y\left[(y - \sqrt{3}a)^2 - 3x^2\right],$$

satisfies the boundary conditions and hence find $w(x, y)$.

Please send any comments or corrections to Dr. C. Beaume.

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